Weibull AFT Regression

Sarah R. Haile

University of Zurich
Institute for Social and Preventative Medicine
Biostatistics Unit

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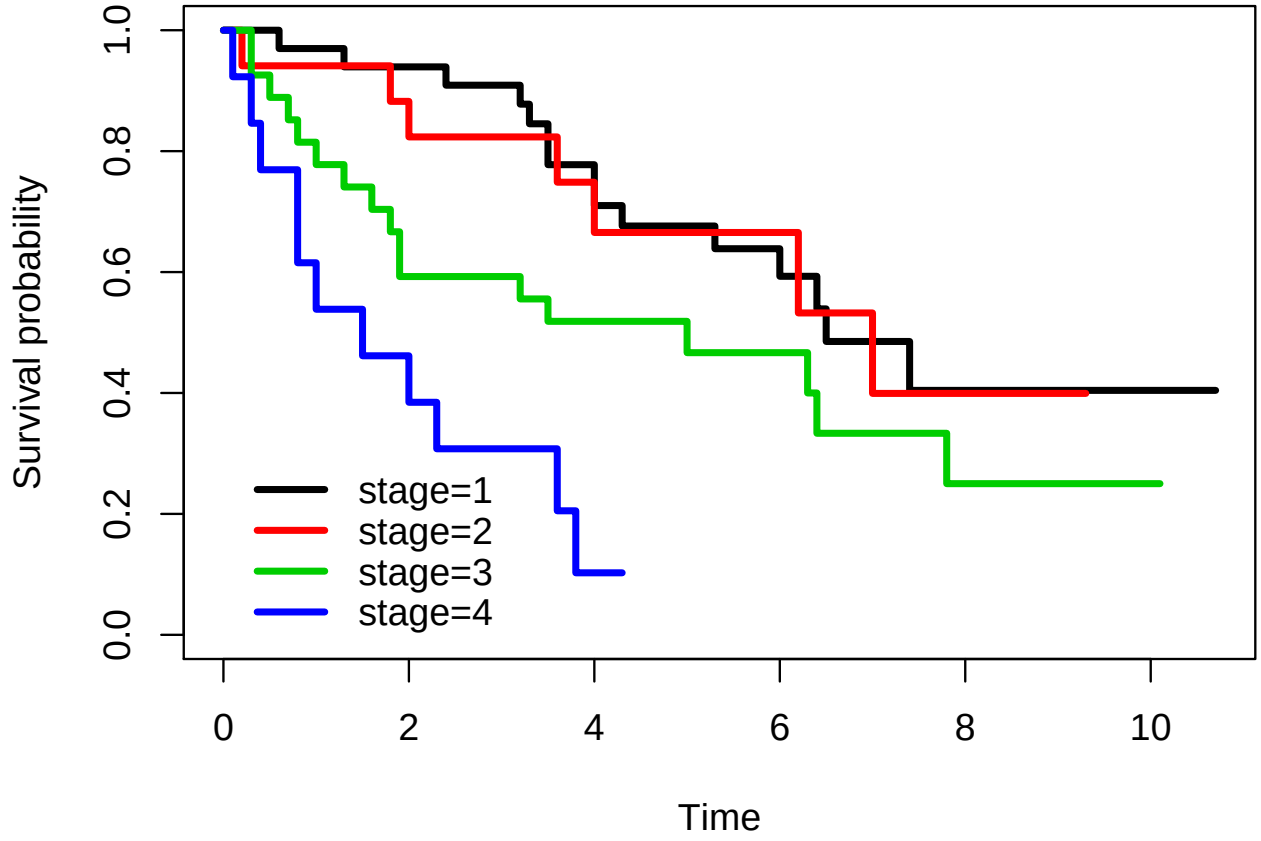
Weibull Distribution

1. Flexible model for survival data
2. Has both Accelerated Failure Time (AFT) and Proportional Hazards (PH) representations
3. Monotone hazard shape

$$h(x|\mathbf{Z}) = (\alpha\lambda t^{\alpha-1})\exp(\beta^T\mathbf{Z})$$

Larynx Data



stage=1]	33	32	31	27	21	17	10	4	2	1	1
stage=2]	17	16	14	12	8	5	4	3	1	0	0
stage=3]	27	21	16	15	13	7	5	4	2	1	0
stage=4]	13	7	5	3	1	0	0	0	0	0	0

survreg

```
[1] "stage"  "time"   "age"    "year"   "death"  "stage2" "stage
```

```
Call:
survreg(formula = Surv(time, death) ~ stage2 + stage3 + stage4 +
    age, data = data, dist = "weibull")
              Value Std. Error      z        p
(Intercept)  3.5288     0.9041  3.903 9.50e-05
stage2      -0.1477     0.4076 -0.362 7.17e-01
stage3      -0.5866     0.3199 -1.833 6.68e-02
stage4      -1.5441     0.3633 -4.251 2.13e-05
age         -0.0175     0.0128 -1.367 1.72e-01
Log(scale)  -0.1223     0.1225 -0.999 3.18e-01

Scale= 0.885

Weibull distribution
Loglik(model)= -141.4   Loglik(intercept only)= -151.1
Chisq= 19.37 on 4 degrees of freedom, p= 0.00066
Number of Newton-Raphson Iterations: 5
n= 90
```

survreg

So, R gives:

- intercept
- (estimates for the covariates)
- log(scale)

These correspond to:

$$\log T = \mu + \gamma^T \mathbf{Z} + \sigma W,$$

where μ is the intercept, γ is the vector of regression parameters, and σ is the scale. Use the following:

$$\alpha = 1/\sigma, \tag{1}$$
$$\lambda = \exp(-\mu/\sigma), \tag{2}$$
$$\beta = -\gamma/\sigma, \tag{3}$$

to get the Weibull model like we are used to.

survreg

- Hazard Ratios:

$$e^{\beta}$$

  (ratio of hazards at time t)
- Event Time Ratio[1]:

$$e^{-\beta/\alpha}$$

  (ratio of times to pth percentile)
- an ETR < 1 indicates that a group experiences events at a faster rate than the baseline, which corresponds (more or less) with a HR > 1

[1] Carroll, 2003 (*Controlled Clinical Trials*)

WeibullReg

WeibullReg and ConvertWeibull

```
> fm <- Surv(time, death) ~ stage2 + stage3 + stage4 + age
> WR <- WeibullReg(fm, data)
> names(WR)
[1] "formula" "coef"    "HR"      "ETR"     "summary"
> signif(WR$coef, 3)
        Estimate     SE
lambda    0.0185 0.0190
alpha     1.1300 0.1380
stage2    0.1670 0.4610
stage3    0.6630 0.3560
stage4    1.7500 0.4150
age       0.0197 0.0142
```

WeibullReg

```
> signif(WR$HR, 3)
```

```
          HR    LB    UB
stage2 1.18 0.479  2.92
stage3 1.94 0.967  3.89
stage4 5.73 2.540 12.90
age    1.02 0.992  1.05
```

```
> signif(WR$ETR, 2)
```

```
         ETR    LB   UB
stage2 0.86 0.390 1.90
stage3 0.56 0.250 1.20
stage4 0.21 0.096 0.47
age    0.98 0.440 2.20
```

coxph

To compare with Cox PH regression

```
> signif(WR$HR, 3)
```

```
          HR    LB    UB
stage2 1.18 0.479  2.92
stage3 1.94 0.967  3.89
stage4 5.73 2.540 12.90
age    1.02 0.992  1.05
```

```
> signif(summary(coxph(fm, data))$conf.int, 3)
```

```
       exp(coef) exp(-coef) lower .95 upper .95
stage2      1.15      0.869     0.465      2.85
stage3      1.90      0.526     0.946      3.82
stage4      5.51      0.182     2.410     12.60
age         1.02      0.981     0.991      1.05
```

WeibullDiag

$$\log H(t|\mathbf{z}) = \log \lambda + \beta^T \mathbf{z} + \alpha \log t$$

```
> WeibullDiag(Surv(time, death) ~ stage, data, labels = c("Stage
+     "Stage II", "Stage III", "Stage IV"))
```